Maths Module 4: Geometry

1. Shapes

1.1 Angles

Practice - *Answers*

i.

a) *a = 48°*   **b)** *b = 106°, c = 74°, d = 74°*   **c)** *e = 111°*   **d)** *f = 60°, g = 120°*   **e)** *c = 51 3/7°*

ii.

a) *a = 35°, b = 50°, c = 50°, d = 95°*   **b)** *e = 68°, f = 68°*   **c)** *g = 149°, h = 125°, i = 55°*

1.2 Triangles

Practice - *Answers*

a) $a = 35°$ **b)** $b = 35°$ **c)** $c = 70°$, $d = 110°$ **d)** $e = 52°$, $f = 85°$
e) $g = 47°$, $h = 101°$, $i = 101°$

1.3 Quadrilaterals

1.4 Interior Angles

Practice - *Answers*

i.
a) isoceles triangle   *b)* parallelogram   *c)* parallelogram   *d)* square

ii.
a) 1440°   *b)* 144°

1.5 Congruence

Practice - *Answers*

i.

a) No. Only the angles are equal. The corresponding sides may not be equal.

b) Yes. PQRS = TUVW. Corresponding sides and angles are equal.

c) Yes. XYZ = NLM.

ii.

a) ABC, YZX

b) ABC, EFD

c) XYZ, QRP

iii. AB = CD, AD = BC and BD = BD. The triangles are congruent since all sides are the same.

iv. BC = BD - CD, DE = CE - CD. But BD = CE, so BC = DE.

CA = AD (given)

Angle BCA = 180° - Angle ACD

Angle ADE = 180° - Angle ADC

But, Angle ACD = Angle ADC, so Angle BCA = Angle ADE

If BC = DE and Angle BCA = Angle ADE then the triangles ABC, ADE are congruent.

v.
Angle ABC = Angle ACB
Angle ABC = Angle ADE
Angle ACB = Angle AED
Angle ADE = Angle AED and ADE is isoceles

Angle AD = Angle AE
So, triangles ACD and ABE are congruent.

vi.
Angle ABE = 180 - 2 x Angle AEB
Angle DBC = 180 - 2 x Angle BDC
But, Angle AEB = Angle BDC
So, Angle ABE = Angle DBC
Angle ABD = Angle ABE + Angle EBD
Angle EBC = Angle DBC + Angle EBD
Therefore Angle ABD = Angle EBC
> *AB = EB*
> *BD = BC*
So, triangles ABD, EBC are congruent

vii.
If we use Pythagoras' Theorem then side AB = 4 cm and side DE = 3 cm, so the triangles are congruent.

1.6 Similar Shapes

Practice - *Answers*

i.

a) *4*   **b)** *2.34*   **c)** *19.8*   **d)** *d = 6.72, e = 4.8*

ii.
a) x = 1.5, y = 3   *b)* 12.8   *c)* x = 3.75, y = 5.25, z = 6.75

iii.
A and C, B and E, D and F

iv.
a) (ii)   *b)* (iii)   *c)* (iii)

v.
a) Two pairs of sides in same ratio, included angle equal. x = 4.72.

b) All corresponding angles are equal. y = 1.8.

c) All corresponding angles are equal. a = 7.68, b = 9.6.

2. Constructions

2.1 Constructing a triangle

2.2 Constructing a regular hexagon

2.3 Constructing perpendiculars

2.3 Bisecting an angle

Practice - *Answers*

*For each question **i.** to **x.** check the students' constructions for accuracy.*
Students can also compare their answers with other students work to check their own accuracy.

vi. Each angle in the triangle is 60°

viii.
a) x = 120°   *b)* y = 30°

x.
b) 8.77 cm

3. Measure and Mensuration

3.1 Introduction

Practice - *Answers*

i.
a) *3.35 m, 3.45 m*   **b)** *4.55 m, 4.65 m*   **c)** *3.35 x 4.55 = 15.24 m², 3.45 x 4.65 = 16.04 m²*

3.2 Perimeter and area of triangles and quadrilaterals

Practice - *Answers*

i.
a) 52.5 cm², 31 cm   *b)* 80 cm², 48 cm

ii.
a) (6x + 8) cm
b) 7
c) 136 cm²

iii.
a) 4 cm
b) 16 cm²

iv. 62500 m²

v.
a) 30 cm²
b) 6 cm²
c) 45 cm²

vi.
a) 4x - 5
b) 8.5

vii.
a) 40 cm²
b) 36 cm²
c) 30 cm²

viii.
a) 70 cm²
b) 8.75 cm

ix. 9.49

x.
a) 105 cm²
b) 14 cm²
c) 66 cm²

3.3 Circumference and area of circles

Practice - *Answers*

i.
a) *31.42 cm, 78.54 cm²*
b) *50.27 cm, 201.06 cm²*
c) *37.70 cm, 113.10 cm²*

ii. *If the circumference and area are the same then: $\pi r^2 = 2\pi r$ which gives $r^2 = 2r$. So $r = 2$*

iii.
a) *2.39 cm*   **b)** *17.94 cm²*

iv. *37.85 cm*

v.
Area = 39.27 cm²
Perimeter = 25.71 cm

vi.
a) *181.70 cm*
b) *1666.19 cm²*

3.4 Volume and area of 3-D shapes

Practice - *Answers*

i.
a) 125 cm³   *b)* 1728 cm³   *c)* 54.872 cm³

ii.
a) 10 cm   *b)* 600 cm²

iii.
a) 180 cm³   *b)* 480.24 cm³

iv.
a) 4 cm   *b)* 352 cm²

v.
a) 504 cm³
b) 396 cm²

vi. 240 cm³

vii. 12 cm

viii. 576 cm³

3.5 Finding the length of an arc of a circle

Practice - *Answers*

i.
  a) *9.77 cm*   **b)** *10.47 cm*   **c)** *7.38 cm*   **d)** *39.10 cm*   **e)** *47.12 cm*   **f)** *60.21 cm*

ii.
  a) *50.13°*   **b)** *80.21°*   **c)** *47.75°*   **d)** *222.82°*   **e)** *73.52°*   **f)** *39.14°*

3.6 Finding the area of a sector of a circle

Practice - *Answers*

i.

a) 22.34 cm²   *b)* 61.09 cm²   *c)* 30.54 cm²   *d)* 45.95 cm²   *e)* 139.63 cm²   *f)* 279.25 cm²

ii. 72.15°

iii. 10.06 cm

3.6 Finding the area of a segment of a circle

Practice - *Answers*

i.

a) 9.06 cm²   *b)* 10.98 cm²   *c)* 0.51 cm²   *d)* 3.22 cm²   *e)* 308.83 cm²   *f)* 192.04 cm²

Note that for e) and f) students have to calculate the area of the segment NOT shown and then subtract this from the total area to find the answer.

ii.

a) 7.16 m

b) 2.99 m²

3.7 Finding volumes and surface areas

Practice - *Answers*

i. 314 cm³, 283 cm²

ii. 156 cm³, 211 cm²

iii. 0.942 m³

3.8 Volume of a pyramid

Practice - *Answers*

i. 960 cm³

ii. 140 cm³

iii. 6.18 cm

3.9 Surface area and volume of a sphere

Practice - *Answers*

i.
a) 2144.7 cm³, 804.25 cm²
b) 1563.5 cm³, 651.4 cm²
c) 3591.4 cm³, 1134.1 cm²
d) $(1/6)\pi x^3$, πx^2

ii. 10.61 cm

iii. 9.67 cm

4. Transformations

4.1 Translation

Practice - *Answers*

i.

*a)* $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$ *b)* $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$ *c)* $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$ *d)* $\begin{pmatrix} -7 \\ 2 \end{pmatrix}$ *e)* $\begin{pmatrix} -6 \\ -3 \end{pmatrix}$
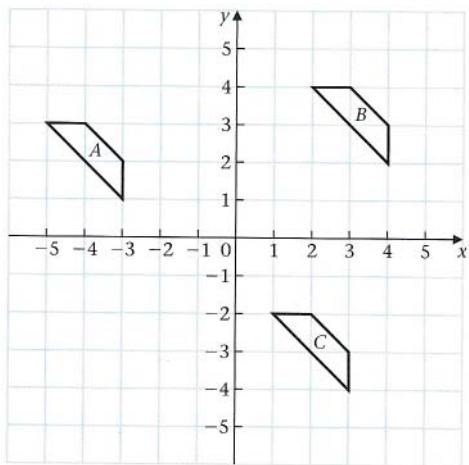
ii.

a) and *b)*
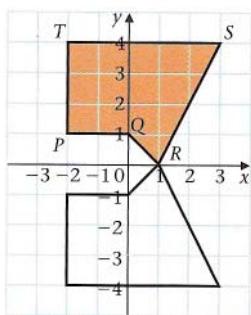
*c)* $\begin{pmatrix} -6 \\ 5 \end{pmatrix}$

4.2 Reflection
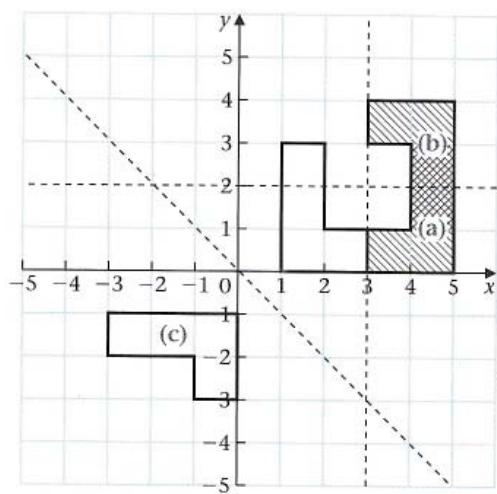
Practice - *Answers*

i.

*ii.* A reflection in the line x = 2.
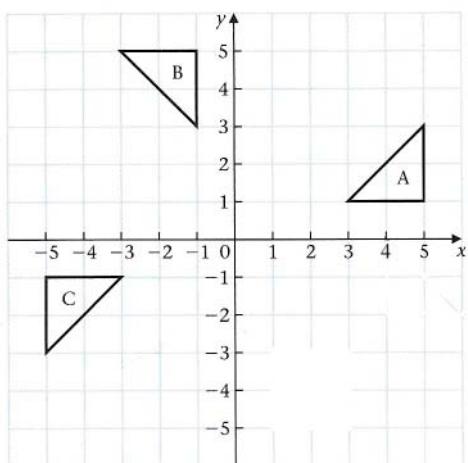
iii.

4.3 Rotation

Practice - *Answers*
i.
a) and *b)*

c) Rotate 180° about the origin
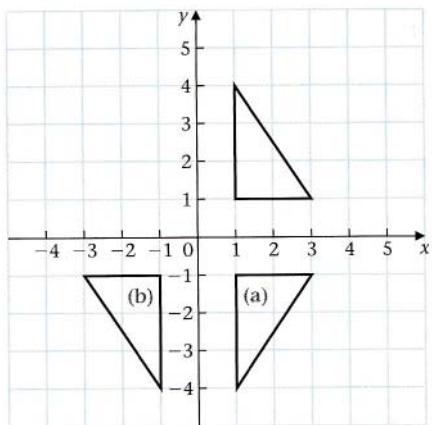
ii. A rotation 180° about the point (0,0).

4.4 Combined transformations

Practice - *Answers*
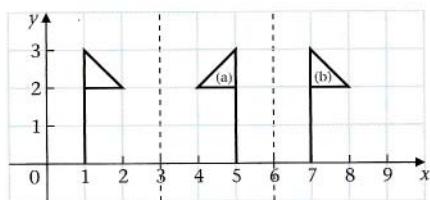
i. a) and **b)**

c) *Rotation 180° about the origin*
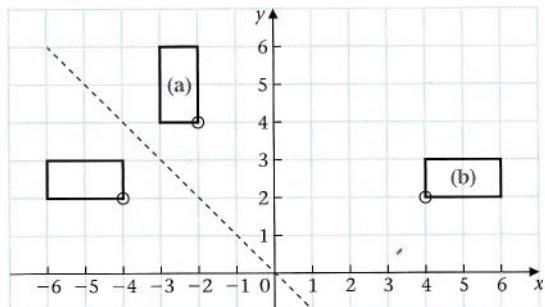
ii. a) and **b)**

c) *Translation* $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$
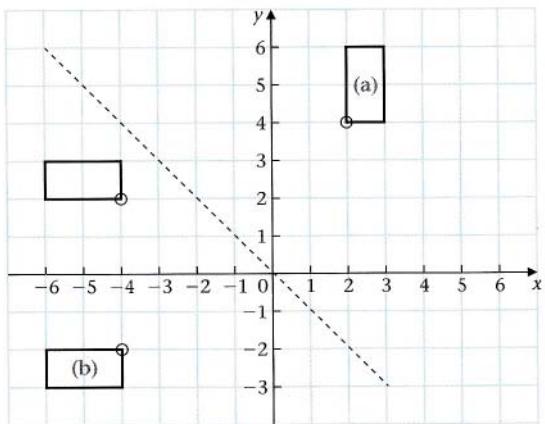
iii. a) and **b)**

c) *Reflection in x = 0 (y-axis)*

iv. a) and **b)**

c) *Reflection in y = 0 (x-axis)*
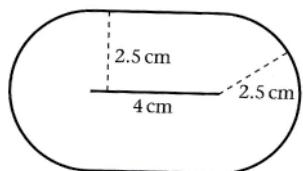
5. Loci

Practice - *Answers*
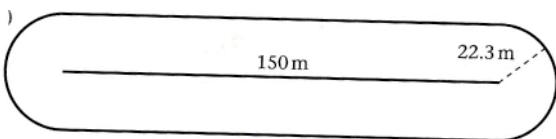
i. Students should draw the perpendicular bisector of AB.

ii.

iii.
a)

b) 440 m (to 3 s.f.)

6. Trigonometry

6.1 Trigonometric ratios

Practice - *Answers*

i.
a) cosine   *b)* cosine   *c)* tangent   *d)* sine   *e)* sine   *f)* tangent
g) sine   *h)* tangent   *i)* cosine

ii. About 9.1 m

6.2 Using trigonometric ratios to find angles

Practice - *Answers*

a) 44.4°   *b)* 38.7°   *c)* 48.2°   *d)* 21.8°   *e)* 65.4°

f) 30°   *g)* 18.4°   *h)* 24.3°   *i)* 52.4°

6.3 Using trigonometric ratios to find the length of sides

Practice - *Answers*

a) 7.66   *b)* 7.71   *c)* 12.99   *d)* 5.34   *e)* 7.52

f) 6.75   *g)* 6.36   *h)* 12.12

a) 44.4°   *b)* 38.7°   *c)* 48.2°   *d)* 21.8°   *e)* 65.4°

Glossary of Keywords

The glossary in the Students' book is a list of all mathematical words that appear in the module. They are given in the order that they appear.

The following short activities are added to this guide to help students remember mathematical vocabulary. They can be used in several ways: to test prior knowledge of a topic, as warm-up activities at the beginning of a lesson or to review what has been learnt at the end of a topic.

Activity 1 - Discuss questions in pairs.

Students are given questions to discuss that relate to a topic.

Example questions -
What is an improper fraction?
How do I change from milligrams to tonnes?
How do I find the perimeter of a square?
What is the commutative law?
What is the order of operations?

Activity 2 - True or false.

Students work in pairs to decide if statements about a topic are true or false.

Example for fractions -
The denominator is the top number in a fraction.
The numerator is less than the denominator in an improper fraction.
Equivalent fractions have the same numerator.

Activity 3 - Give an explanation.

Students work in pairs to prepare a short explanation to questions. Ask some students to give their explanation to the class.

Examples -
Explain how to change from a mixed number to an improper fraction.
Explain how to calculate: $(2 + 3) \times (7 - 42)3$
Explain the mistake in this statement:
Explain what a negative number is.

Activity 4 - Brainstorming

Write a topic on the board and ask students what they know about the topic. Write their answers on the board.

Activity 5 - What's the topic?

Write words linked to a topic on the board and ask students if they can guess the topic.

Assessment

This is assessment covers most of the topics in this module and should give you an idea of how much the students have understood. It is recommended that you give it as a class test, with some time for review and revision beforehand.

Part 1 - Answers

Each question in part 1 is worth 1 mark

a) Vectors   **b)** Perpendicular   **c)** Scalene   **d)** Congruent   **e)** Bisector   **f)** Enlargement

g) Circumference   **h)** Locus   **i)** Rotation   **j)** Supplementary

Total for part 1: 10 marks

Part 2 - Answers

The total mark for each question is given on the right hand side of the page. Tell students to use $\pi = 3.14$ when needed. If there are no calculators then students can leave their answers to 16 b), 17 b) and 18 as trigonometric formulae.

1. AF, BD, CH, EG   *4 marks*

2. a) y = 2.7   *b)* q = 4.2, r = 5.88   *c)* x = 6   *d)* p = 25.2   *5 marks*
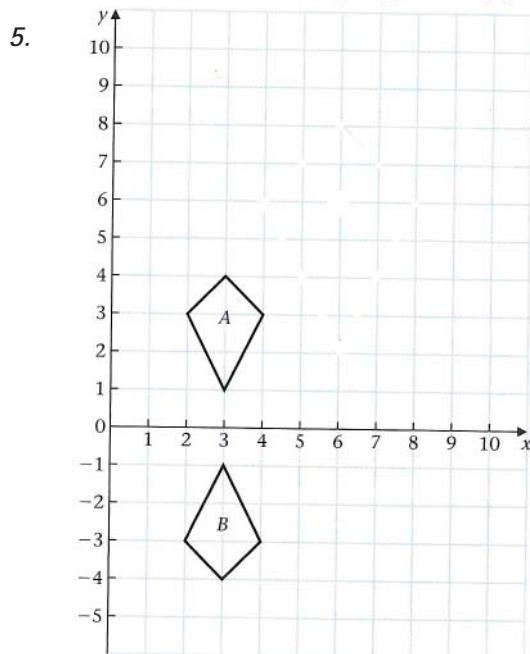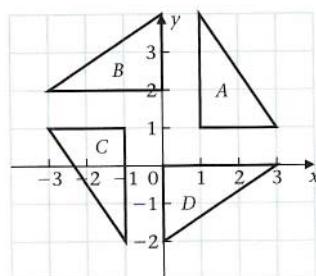
3.
a) Alternate angles and vertically opposite angles are equal so interior angles all match   *3 marks*
b) x = 1.6, y = 4.48

*4.*
*a)* AXY and ABC   *b)* One interior angle is shared, the other two are corresponding   *4 marks*
c) 1.9125   *d)* 1.788

5.
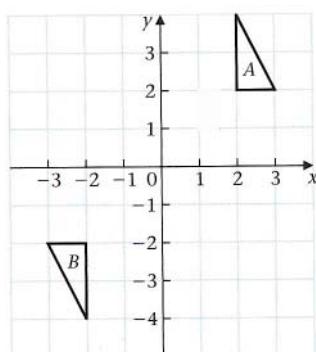
6.
a)   *2 marks*

                                                                                                          4 marks

b) Rotation 180° about (0,1)

7.

                                                                                      2 marks

8.
a)   *3 marks*
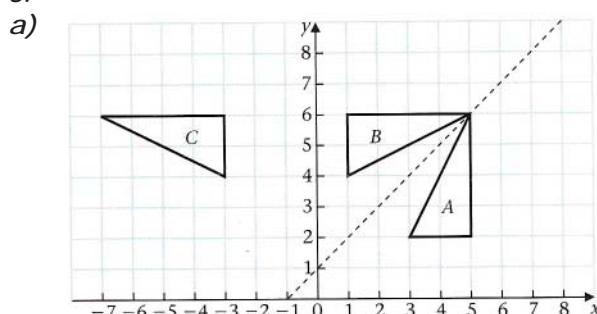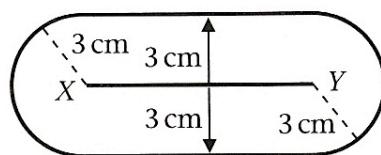
b) y = x + 1
c) 90° clockwise rotation about (-1,0)

9.

   *2 marks*

10.
Check the students' constructions for accuracy.   *4 marks*

11.
a) 86.625 m³   *b)* 119.5 m²   *2 marks*

12. a) 317.08 m   *b)* 5963.5 m^2 *3 marks*

13. 50 cm^2 *2 marks*

14. 154.06 cm^2 *2 marks*

15. 600 cm^3, 660 cm^2 *3 marks*

16. a) 12.37 m   *b)* 72.1°   *3 marks*

17. a) 8.15 m   *b)* 38.7°   *3 marks*

18. 7.71 m   *1 mark*

19. a) 6.76 cm   *b)* 6.98 cm   *c)* 27.93 cm^2 *d)* 3.41 cm^2 *4 marks*

20. 9048 cm^2, 2413 cm^2 *3 marks*

21. 268.08 cm^3 *2 mark*

22. 1120 cm^3 *2 mark*

23. 9366.4 cm^3 *2 mark*

Total for part 2:
65 marks

Final total:
75 marks